Although the fast filter worked well for cases where more concentration measurements were available (for example, Soliman and Ray, 1978), for the results shown here, the fast filter did not converge. This is not surprising in case 2, where there are no concentration measurements, since one would not expect the fast concentration dynamics to be observable without direct concentration measurements. For case 1, it is likely that a single exit concentration sensor is not sufficient for system observability.

The slow filter performance, shown in Figures 1 and 2, was quite acceptable in both cases. Note that both the temperature and the quasi-steady-state concentration estimates converge quickly to the true profiles in case 1, while for case 2 the estimates are less accurate but reasonable.

This example suggests that slow state estimates will perform acceptably, even in the case of no direct concentration measurements, but will perform significantly better with one or more composition sensors. The fast state estimator, which would be needed to follow fast concentration dynamics, requires more than one concentration sensor for good performance. However, in many applications the fast estimator would not be required, particularly if feed concentration were not a control variable.

Presently work is underway in our laboratory to test these algorithms on-line in real time with pilot scale reactors.

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NOTATION

B = dimensionless heat of reaction
Da_0 = reference Damköhler number
H = dimensionless heat transfer coefficient
P = differential sensitivity filter parameter
Pe_H = Peclet number for heat transfer
Pe_M = Peclet number for mass transfer
Q = filter parameter
t = dimensionless time
x_1 = dimensionless reactant temperature
x_{1c} = dimensionless coolant temperature
x_2 = dimensionless reactant concentration
y_1 = temperature measurement
y_2 = concentration measurement
z = dimensionless length along the reactor

Greek Letters

γ = dimensionless activation energy
η = measurement error
ξ = model error
ϵ = ratio of reactor residence time and thermal time constant
τ = t/ϵ

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The Expansion of a Fluidized Bed Containing Biomass

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Much interest has been shown recently in treating wastewater by passing it through a fluidized bed of particles on which a bacterial film is growing. The possibilities of such systems were first demonstrated by Freidman et al. (1971) in their work on the importance of bacterial growth in beds of activated carbon. Beds of coal and sand have

since been proposed for denitrification (Jeris and Owen, 1975), secondary treatment of domestic sewage (Jeris et al., 1977), and the removal of phenolic compounds from a concentrated coal processing waste (Lee and Scott, 1977). Their major advantage over conventional biofilm reactors (the trickling filter and biodisc processes) is their enormous surface area of film, which gives large removal rates in a relatively small reactor volume.

The growth of a bacterial film in a fluidized bed will not clog it because the bed is able to expand to accommodate the extra volume. The resulting expansion is so large that the bed height becomes an important operating parameter. The objective of this paper is to show how the height is related to the amount of biomass in the bed. This relation can be used both in the design stage to predict the bed height and during operation to infer the quantity of biomass in the bed from its height. Since the equations apply equally to adsorptive particles, the latter approach also provides an easy method for detecting and quantifying bacterial growth in fluidized-bed, activated carbon adsorption columns.

The general relation is found as follows. Consider a small increment of bed height, of volume dV, containing an incremental proportion dy of the total clean particle volume. The porosity in the element is by definition

$$\epsilon = 1 - (1 + x)v\frac{dy}{dV} \tag{1}$$

Rearranging and integrating, we get

$$\frac{V}{v} = \frac{H}{H_c(1 - \epsilon_c)} = \int_0^1 \frac{1 + x}{1 - \epsilon} dy \tag{2}$$

The dependence of the porosity (ϵ) on the amount of film (x) is derived as follows. For constant superficial velocity, the correlation of Richardson and Zaki (1954) can be written

$$\frac{\epsilon}{\epsilon_c} = \left(\frac{u_c}{u}\right)^{1/n} \tag{3}$$

Note that the value of the constant n is assumed to be unchanged by the growth of the film.

The particle settling velocity is affected by the growth of a bacterial film in two counteracting ways. Settling velocity is proportional to the buoyant density, which is reduced by film growth, and to the square of the particle size, which is increased. The net effect is a small change given by

$$\frac{u}{u_c} = \frac{1 + Ax}{(1 + x)^{1/3}} \tag{4}$$

Note that this equation is based on Stoke's law and so will have to be modified for particles significantly heavier than those used in this work.

Equations (3) and (4) give an expression for ϵ in terms of x, which can be substituted into (2).

The remaining barrier to the integration of Equation (2) is that the film is not distributed evenly through the bed. The amount of film on any particular particle depends on the number of bacteria captured by the particle during seeding of the bed; the bacterial growth rate, which depends on the average substrate concentration to which the particle has been exposed since seeding, which depends in turn on the average vertical position of the particle in the bed and thus on the solids mixing pattern; and the number of bacteria washed or knocked off by attrition between particles. Since seeding, mixing, and interparticle collision are random processes, they leave more film on some particles than others. The resulting distribution of film over the particles can be described by a distribution function $g(x)$. However, little can be said a priori about the form of this function, except that it must be defined for values of x between zero and some maximum value (x_{max}).

An additional difficulty is that the bed, like all fluidized beds, tends to stratify vertically according to the settling velocity of the particles. Observation of a bed of coal particles clearly shows that particles heavily coated with film tend to rise to the top of the bed. [Note that this indicates that film growth reduces settling velocity; a binomial expansion of Equation (4) shows that this happens only if $A < 1/3$]. The eventual vertical distribution of biomass in the bed depends on the extent to which this tendency to stratify is balanced by mixing of the particles. Since neither solids mixing nor stratification in liquid fluidized beds are well-understood phenomena, only the extreme cases are analyzed here.

COMPLETE SOLIDS MIXING (NO STRATIFICATION)

If mixing is complete, the average film thickness will be the same in all parts of the bed. When $x = \bar{x}$ everywhere, Equations (2), (3), and (4) give

$$\frac{H}{H_c} = \frac{(1 - \epsilon_c)(1 + \bar{x})}{1 - \epsilon_c\left[\frac{(1 + \bar{x})^{1/3}}{1 + A\bar{x}}\right]^{1/n}} \tag{5}$$

COMPLETE STRATIFICATION (NO MIXING)

In this case, no particle is below another which has a higher settling velocity. Clearly (for small A), the film coverage now increases uniformly with distance up the bed. At any point in the bed the coverage is x, and the proportion of clean particle volume which is below the point is y. Given the definition of complete stratification, the particles below this point must all have coverage less than x. But the proportion of the particles which have coverage less than x is given by the cumulative distribution function. Thus

$$y = \int_0^x g(x)\, dx \tag{6}$$

Equations (2) and (6) give

$$\frac{H}{H_c} = (1 - \epsilon_c)\int_0^{x_{max}} \frac{(1 + x)}{1 - \epsilon} g(x)\, dx \tag{7}$$

Typical results for complete mixing and complete stratification with uniform distribution [$g(x) = 1/2\bar{x}$ for $2\bar{x} > x > 0$] are shown in Figure 1. Clearly, the relation between bed height and film coverage is approximately linear and is not greatly affected by the solids mixing condition, particularly for small x. This happens because the change in settling velocity produced by film growth is small. Since n is approximately 4.5, Equation (3) predicts an even smaller change in porosity (less than 4% for $x < 1$). A linear approximation to the function $(1 + x)/(1 - \epsilon)$ is therefore quite accurate. Taking the best linear approximation for the range 0 to x' and substituting in (7), we get

$$\frac{H}{H_c} = 1 + (1 + B)\bar{x} \quad B = \frac{\epsilon_c D}{3(1 - \epsilon_c)}\left[3 + x'\left(2\right.\right.$$

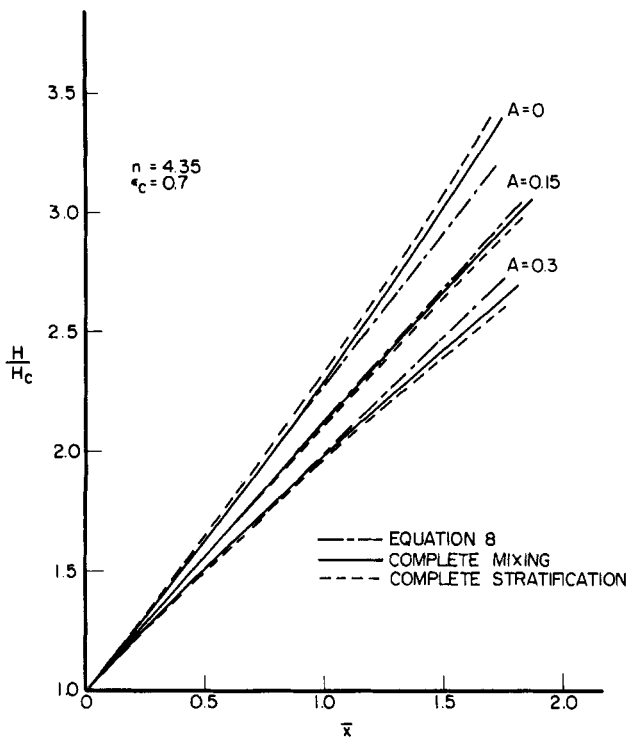

Fig. 1. The predicted variation of bed height with film volume.

$$+ D\left(\frac{1+\epsilon_c}{1-\epsilon_c}\right) - \frac{1-3A^2}{1-3A}\Bigg)\Bigg] \qquad (8)$$

$$D = \frac{1-3A}{3n}$$

This expression is completely independent of both the solids mixing condition and the distribution function $g(x)$. Figure 1 (drawn for $x' = 1$) shows that it gives a reasonable approximation to the predictions of Equations (5) and (7). It can therefore be adopted as a practical working expression.

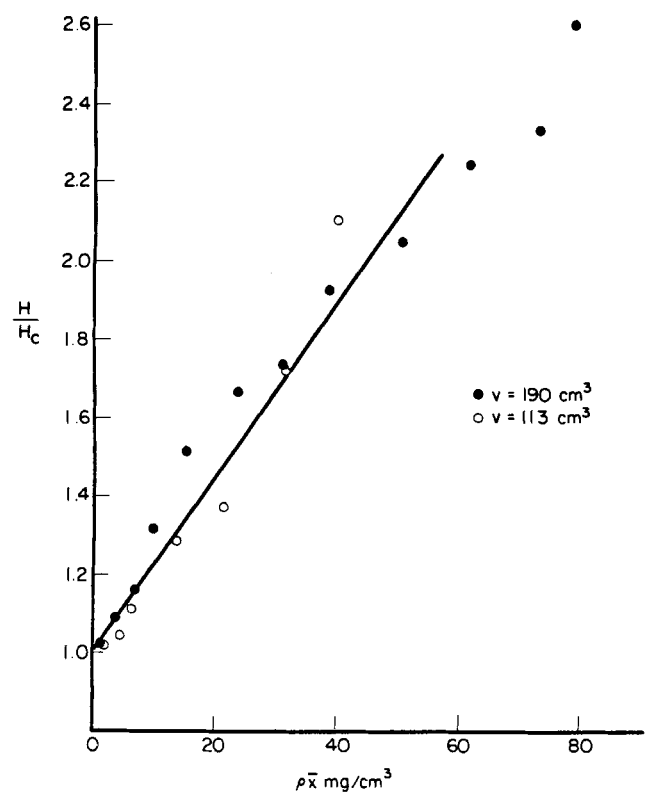

Fig. 2. Comparison of experimental data with Equation (8).

The major source of uncertainty in the analysis presented here is that the film density may not be a true constant. Hoehn and Ray (1973) have shown that the dry weight per unit volume of a thin film increases with film thickness. The film density may follow a similar pattern. Also, in concentrated denitrifying systems, the nitrogen formed may come out of solution and form bubbles in the film. Whether these effects are strong enough to warrant inclusion in the model can only be determined experimentally.

EXPERIMENTAL WORK

Experiments were conducted on beds of 40 × 45 mesh coal (specific gravity 1.298) in a Plexiglas column 44.5 mm in diameter. The influent stream contained 125 mg/l of Dextrose, 475 mg/l of sodium nitrate (to allow the growth of denitrifying bacteria after the dissolved oxygen is depleted), and a phosphate buffer in dechlorinated tap water. The bed was seeded by filling the column with this mixture, plus a bacterial seed culture, and recirculating it through the bed for several days. (The seed contained a mixed culture of denitrifying bacteria developed from sewage by repeated culturing in a Dextrose substrate.) During this period, the variation of bed height with flow rate was studied in order to determine the value of the parameter n. At the end of the seeding period, the influent stream was turned on, and the recirculation was maintained so that the column approximated a backmix reactor. The total carbon concentrations in the influent and effluent streams were measured at regular intervals using a Beckman model 915A TOC analyzer.

Verification of Equation (8) clearly requires an independent measure of the quantity of biomass in the bed. This is obtained from the total carbon measurements as follows. As bacteria grow in films on the coal, they remove organic carbon from solution as substrate. However, they also return carbon to solution as carbon dioxide, organic metabolic products, and cells or cell fragments that are washed off the film. The only carbon removed permanently from solution is that which is incorporated into the biomass. The conservation equation for total carbon over the seeding period is therefore (assuming the coal is initially sterile)

$$\Delta TC_o = \rho v' \bar{x}_o \qquad (9)$$

A conservation equation may also be written for the backmix reactor and integrated with Equation (9) as an initial condition. This gives the required equation:

$$\rho\bar{x} = \frac{1}{v'}\left(\Delta TC + \frac{1}{\tau}\int_o^t \Delta TC\, dt\right) \qquad (10)$$

[Note that Equations (9) and (10) are exact only when the total carbon concentration is very much less than ρ.] Two runs were completed using a superficial velocity of 0.543 cm/s (giving $\epsilon_c = 0.795$) and clean particle volumes $v = 190$ and 113 cm³. During the first run, the column was inclined 0.4 deg from the vertical. This was sufficient to cause visible recirculation of the bed solids, with a recirculation time of the same order of magnitude as the liquid residence time. Thus, the condition of complete solids mixing was approximated. In both runs, the bed height increased steadily after an initial small drop of 4 to 5%, which was probably caused by agglomeration or attrition of the particles. The minimum observed bed height was used as the value of H_c.

The results are shown in Figure 2. The agreement

between the data and the single straight line is encouraging, particularly in view of the different solid mixing conditions in the two runs. Some scatter in the data is to be expected, principally because of the large percentage error in the ΔTC measurement when ΔTC is small. Estimation of the film density as 1.05 g/cm^3 gives $A = 0.16$ which, with the slope of the line in Figure 2 and the measured value of n (4.55), gives $\rho = 53\,600$ mg/l. Thus, the abscissa of Figure 2 goes to approximately $\bar{x} = 1.4$. The slight curvature shown by the data from the first run at large $\bar{x}$ is as would be expected from the model (see Figure 1).

CONCLUSIONS

The growth of biomass in a liquid fluidized bed causes it to expand in a predictable way. In general, the expansion will depend on the way the biomass is distributed over the bed particles and on the solids mixing condition. However, it can be approximated over a wide range by a linear function which is independent of these factors. This function should prove useful both for inferring the quantity of biomass in a bed and for predicting the bed height.

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NOTATION

A = buoyant density of bacterial film/buoyant density of clean particle
$g(x)$ = distribution function of bacterial film
H = bed height
n = exponent in the Richardson-Zaki correlation
ΔTC = inlet total carbon concentration-outlet total carbon concentration
t = time, hr
u = particle settling velocity
v = clean particle volume
V = bed volume
v' = particle volume/liquid volume in reactor containing no bacterial film
x = film volume/clean particle volume
$\bar{x}$ = mean value of $g(x)$
y = fraction of clean particle volume below a point in the bed
ϵ = bed porosity
ρ = mg carbon/l bacterial film
τ = space time of reactor containing clean particles

Subscripts

c = bed of clean particles
o = condition at time zero
max = maximum value

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Effectiveness of Bidisperse Catalysts

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Many of the supported porous catalysts have a bidisperse pore structure. Such catalyst pellets are formed by the agglomeration of porous particles. Pores within these particles are usually called micropores, and pores between the agglomerated particles are called macropores. In such catalysts, most of the active centers lie within the particles in the micropore region. Hashimoto et al. (1976) presented a method to predict the effective diffusivities both in the macro and micropore regions of bidisperse catalysts. It has also been shown by Uyanik (1977) that both macro and micropore diffusivities can be determined by the method of single pellet chromatography which is originally developed by Doğu and Smith (1975). Diffusion and adsorption in bidisperse porous catalysts are studied by Hashimoto and Smith (1974). Wakao and Smith (1964) derived an expression for the effective diffusivity for diffusion in bidisperse porous catalyst pellets under reaction conditions. They showed that this diffusivity is a function of the effectiveness factor of the microporous particles. They also discussed the need of knowledge of the pore size distributions to predict the effectiveness factors. Silveston and Hashimoto (1971) incorporated the pore size distribution into the evaluation of effectiveness factors. Mingle and Smith (1961) derived the microeffectiveness factors for several pore distribution functions for a nonisothermal pellet. Carberry (1962) evaluated the effectiveness factor for the reversible first-order reaction. Diffusion and reaction in porous catalysts are reviewed by Aris (1975) in detail.

The effectiveness of bidisperse catalysts depend upon the rate of diffusion of reactants and products both in the macro and micropore regions as well as the rate of reaction. This implies that the prediction of the effectiveness factor of such catalysts from a single parameter, namely, the Thiele modulus (Petersen, 1965), may give erroneous results.